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Can Hawking temperatures be negative ?

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Abstract

It has been widely believed that the Hawking temperature for a black hole is *uniquely* determined by its metric and *positive*. But I find that this is "not" true in the recently discovered black holes which include the exotic black holes and the black holes in the three-dimensional higher curvature gravities. I show that the Hawking temperatures, which are measured by the quantum fields in thermal equilibrium with the black holes, are *not* the usual Hawking temperature but the *new* temperatures that have been proposed recently and can be *negative*. The associated new entropy formulae, which are defined by the first law of thermodynamics, versus the black hole masses show some genuine effects of the black holes which do not occur in the spin systems. Some cosmological implications are noted also.

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I. INTRODUCTION

A black hole is defined by the existence of the non-singular event horizon r_+, which is the boundary of the region of space-time which particles or photons can escape to infinity, *classically*. Bekenstein has shown that the black hole can be considered as a "closed" thermodynamical system with the temperature, proportional to the surface gravity κ_+, and the chemical potentials, proportional to the angular velocity Ω_+ or electric potential Φ_+, if there is, at the horizon [1]. The argument was based on the Hawking's area (increasing) theorem [2] and the black-hole analogue of the first law with the temperature $T_+ \propto \kappa_+$, which is "non-negative", and the entropy $S \propto \mathcal{A}_+$ for the horizon area $\mathcal{A}_+$, which is "non-decreasing", i.e., satisfying the second law of thermodynamics, due to the area theorem, as well as being non-negative. Later Hawking found that the black hole can radiate, from the quantum mechanical effects, with the thermal temperature $T_+ = \hbar\kappa_+/2\pi$ in accordance with the Bekenstein's argument [3] [I am using units in which $c = k_B = 1$]; in this case, the black hole would not be a closed system anymore but interacting with its environments such as the generalized second law needs to be considered [3, 4].

There is an alternative approach to compute the Hawking temperature by identifying $\hbar/T_+ = 2\pi/\kappa_+$ as the periodicity of the imaginary time coordinate which makes the metric regular at the horizon [5] and this approach has been widely accepted; No counter examples for this approach have been known so far, as far as I know. Now, since the surface gravity κ_+ at the horizon can be computed from the metric unambiguously, the Hawking temperature in this approach is *uniquely* determined also. This would be the origin of the widespread belief that the Hawking temperature *be* uniquely determined by the metric in *any* case. And also, it has been widely believed that the Hawking temperature *be* positive as in the Bekenstein's original argument [1]. Actually this belief has been closely related to the "positive mass theorems" for black holes and the fact that the mass is grater than the modulus of the charge, if there is [6].

In this Letter I show that this belief is "not" true in the recently discovered black holes which include the exotic black holes and the black holes in the three-dimensional higher curvature gravities.

II. NEW HAWKING TEMPERATURES FROM THERMODYNAMICS

In the spin systems the temperature can be negative due to the upper bound of the energy level [7]. Recently a number of black hole solutions which have similar upper bounds of the black hole masses have been discovered [8, 9, 10, 11, 12]. I have argued that the Hawking temperatures for these systems are not given by the usual formula $T_+ = \hbar\kappa_+/2\pi$ [8, 9, 10], which is non-negative, but by new formulae, which *can* be *negative* depending on the situations [11, 12]. The argument was based on the Hawking's area theorem and the second law. This has been found to agree completely with the CFT analysis, being related to the AdS/CFT correspondence. In this section let me briefly introduce the black hole solutions and the thermodynamical arguments for the new Hawking temperatures which differ from the usual formula and can be negative.

A. The exotic BTZ black holes

An exotic BTZ black hole is characterized by the following properties

a. The metric is the same as the BTZ black hole solution [8, 10, 11, 12], which is given by [13],

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (d\phi + N^{\phi} dt)^2 \tag{1}$$

with

$$N^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{l^2 r^2}, \quad N^{\phi} = -\frac{r_+ r_-}{l r^2}, \tag{2}$$

or modulus a 2-sphere [9]. Here, r_+ and r_- denote the outer and inner horizons, respectively.

b. The mass and angular momentum are *completely* interchanged from the "bare" ones $m, \ j$ as

$$M = xj/l, \quad J = xlm \tag{3}$$

with an appropriate coefficient x; $x = 1$ in Ref. [8], x is a fixed value of $U(1)$ field strength in Ref. [9], and x is proportional to the coefficient of a gravitational Chern-Simons term in Refs. [10, 11]. Here, the bare mass and angular momentum for the BTZ black hole are given by

$$m = \frac{r_+^2 + r_-^2}{8Gl^2}, \quad j = \frac{2r_+ r_-}{8Gl}, \tag{4}$$

respectively, with a cosmological constant $\Lambda = -1/l^2$. The radii of the horizons are given by

$$r_{\pm} = l\sqrt{4Gm\left[1 \pm \sqrt{1 - (j/ml)^2}\right]} \tag{5}$$

and it is clear, from this, that the bare parameters, which are positive semi-definite, satisfy an inequality

$$m \geq j/l \tag{6}$$

in order that the horizons exist (the equality for the extremal black hole with $r_+ = r_-$). The remarkable result of (3) is that

$$M^2 - J^2/l^2 = x^2[j^2/l^2 - (m)^2] \leq 0 \tag{7}$$

for *any* non-vanishing x, which shows an upper bound for the mass squared M^2 and a saturation for the extremal bare parameters, i.e., $m = j/l$.

Now, given the Hawking temperature and angular velocity for the event horizon r_+ of the metric (1), following the usual approach [5],

$$T_+ = \frac{\hbar\kappa}{2\pi}\bigg|_{r_+} = \frac{\hbar(r_+^2 - r_-^2)}{2\pi l^2 r_+}, \quad \Omega_+ = -N^{\phi}\Big|_{r_+} = \frac{r_-}{l r_+} \tag{8}$$

with the surface gravity $\kappa = \partial N^2/2\partial r$, the black hole entropy has been identified as

$$S = x\frac{2\pi r_-}{4G\hbar}, \tag{9}$$

which satisfies the first law

$$\delta M = \Omega_+ \delta J + T_+ \delta S \tag{10}$$

but depends on the *inner* horizon area $\mathcal{A}_- = 2\pi r_-$ [8, 9, 10], rather than the outer horizon's $\mathcal{A}_+ = 2\pi r_+$. But there is *no* physical justification of this since the second law is not guaranteed [11, 12] (for an explicit demonstration, see Ref. [14]). Rather, I have recently proposed another entropy formula which does not have this problem

$$S_{new} = |x|\frac{2\pi r_+}{4G\hbar}, \tag{11}$$

in accordance with the Bekenstein's original proposal [1]. Then, it is quite easy to see that this does satisfy the second law since the metric (1) satisfies the Einstein equation in vacuum, regardless of the details of the actions,

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - \frac{1}{l^2}g^{\mu\nu} = 0 \tag{12}$$

: The Raychaudhuri's equation gives the Hawking's area theorem for the outer horizon $\delta\mathcal{A}_+ \geq 0$, i.e., $\delta S_{new} \geq 0$ since this vacuum equation satisfies the null energy condition trivially [2, 3, 4]; this can be also proved by considering a "quasi-stationary" process which does *not* depend on the details of the gravity theory [14, 15]. These results are closely related to the fact that $dr_+/dm > 0,\ dr_-/dm \leq 0$ for any (positive) m and j (equality for $j = 0$) since these describe the rates of the area changes under the positive energy matter accretion.

One interesting consequence of the new identification (11) is that I need to consider the rather unusual Hawking temperature and angular velocity ($\epsilon \equiv sign(x)$)

$$T_- = \epsilon \left.\frac{\hbar\kappa}{2\pi}\right|_{r_-} = \epsilon\frac{\hbar(r_-^2 - r_+^2)}{2\pi l^2 r_-}, \quad \Omega_- = -N^\phi\Big|_{r_-} = \frac{r_+}{l r_-}, \tag{13}$$

respectively [11, 12][1], such as the first law, as well as the manifest second law, be satisfied also

$$\delta M = \Omega_- \delta J + T_- \delta S_{new}. \tag{14}$$

The (positive) numerical coefficient of T_- in (13) is not determined from the thermodynamical arguments but needs some other independent identifications: This has been confirmed *indirectly* in the CFT analysis, by computing the entropy (11) independently [11, 12]; but in this Letter I confirm this in a more traditional way by identifying the Hawking temperature *directly* from the Green function analysis for a quantum field. However, it is important to note that, regardless of the numerical ambiguity, the temperature T_- becomes "negative"

[1] This does not mean, of course, that one needs an observer sitting on the inner horizon r_- to measure T_- and Ω_-, as it does not for $T_+,\ \Omega_+$.

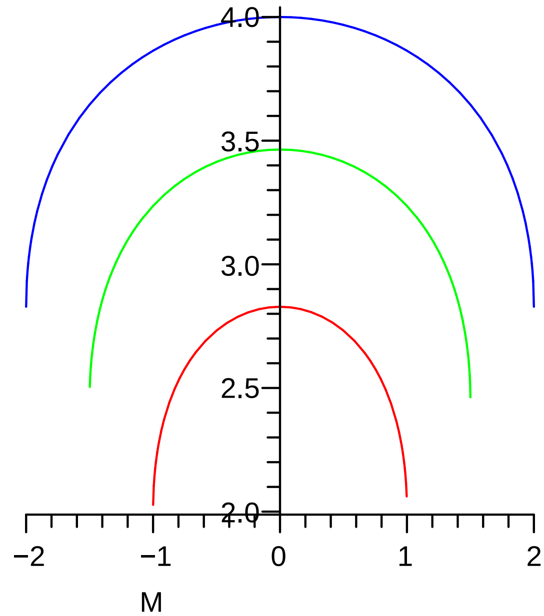

FIG. 1: The normalized entropies $S_{new}(|x|2\pi l/4G\hbar)^{-1}$ vs. M for various values of $|J|/l$=1 (red), 1.5 (green), 2 (blue) [bottom to top] ($l = G = |x| = 1$).

always for $x > 0$. This can be easily understood from the existence of the upper bound of mass $M \leq J/l$ with positive M and J, as in the spin systems [7]. Whereas the temperature T_- becomes positive for $x < 0$ due to the lack of an upper bound, i.e., $J/l \leq M$ with *negative* M and J. These behaviors can be nicely captured in the entropy, as a function of M and J (Fig.1), using (3) and (5):

$$S_{new} = |x| \frac{2\pi l}{4G\hbar} \sqrt{(4GJ/xl) \left[1 + \sqrt{1 - (Ml/J)^2} \right]}. \tag{15}$$

Here I note that the curves in Fig.1 are symmetric about $M = 0$, as in the spin systems: By the definition of the temperature $1/T = (\partial S/\partial M)_J$, I have $T_- < 0$ on the right hand side ($x > 0$), whereas $T_- > 0$ on the left hand side ($x < 0$); the two temperatures $T_- = \pm\infty$ correspond to the same temperature for a vacuum with $M = 0$. But note also that the entropy does *not* vanish at the energy boundary $M = J/l$, i.e., extremal black hole and this would be inherent to black hole systems which does not occur in spin systems [16].

B. The BTZ black hole with higher curvatures

The (2+1)-dimensional gravity with the higher curvature terms and a " bare " cosmological constant $\Lambda = -1/l^2$ can be generally described by the action [omitting some boundary terms]

$$I_g = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(f(g^{\mu\nu}, R_{\mu\nu}, \nabla_\mu) + \frac{2}{l^2} \right), \tag{16}$$

where $f(g^{\mu\nu}, R_{\mu\nu}, \nabla_\mu)$ is an *arbitrary* scalar function constructed from the metric $g^{\mu\nu}$, Ricci tensor $R_{\mu\nu}$, and the covariant derivatives ∇_μ [17, 18]. The equations of motion are

$$\frac{\partial f}{\partial g_{\mu\nu}} - \frac{1}{2} g^{\mu\nu} f - \frac{1}{l^2} g^{\mu\nu} = t^{\mu\nu}, \tag{17}$$

where $t^{\mu\nu}$ is given by

$$t^{\mu\nu} = \frac{1}{2}(\nabla^\nu \nabla^\alpha P_\alpha{}^\mu + \nabla^\mu \nabla^\alpha P_\alpha{}^\nu - \Box P^{\mu\nu} - g^{\mu\nu} \nabla^\alpha \nabla^\beta P_{\alpha\beta}) \tag{18}$$

with $P_{\alpha\beta} \equiv g_{\alpha\mu} g_{\beta\nu} (\partial f / \partial R_{\mu\nu})$.

In the absence of the higher curvature terms, the BTZ solution (1) is the *unique* black hole solution in vacuum. Whereas, even in the presence of the generic higher curvature terms, the BTZ solution can be still a solution since the *local* structure would be "unchanged" by the higher curvatures: Actually $t_{\mu\nu} = 0$ for the BTZ solution and the only effects are some "re-normalization" of the bare parameters $l, r_\pm$, and the Newton's constant G, giving the Einstein equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \frac{1}{l_{ren}^2} g^{\mu\nu} = 0 \tag{19}$$

in the renormalized frame [12, 19]. The renormalized cosmological constant $\Lambda_{ren} = -1/l_{ren}^2$ depends on the details of the function f, but the renormalized Newton's constant is given by

$$G_{ren} = \hat{\Omega}^{-1} G \tag{20}$$

with

$$\hat{\Omega} \equiv \frac{1}{3} g_{\mu\nu} \frac{\partial f}{\partial R_{\mu\nu}}, \tag{21}$$

which is constant for any constant-curvature solution [18]. Now, due to the renormalization of the Newton's constant, the original mass and angular momentum in (4) are modified as

$$M = \hat{\Omega} m, \quad J = \hat{\Omega} j, \tag{22}$$

respectively, by representing m and j as those in the renormalized frame $m = \frac{r_+^2 + r_-^2}{8 G l_{ren}^2}$, $j = \frac{2 r_+ r_-}{8 G l_{ren}}$, with the renormalized parameters $l_{ren}, r_\pm$, but still with the bare Newton's constant G, such as $m \geq j/l$ is valid still. Here it is important to note that $\hat{\Omega}$ is *not* positive definite[2] such as the usual inequality for the mass and angular momentum would not be valid in general,

$$M - J/l = \hat{\Omega}(m - j/l) \tag{23}$$

but depends on the sign of $\hat{\Omega}$: $M \geq J/l$ for $\hat{\Omega} > 0$, but $M \leq J/l$ for $\hat{\Omega} < 0$.

[2] This means a negative Newton's constant but there is no a priori reason to fix the sign in three dimensional gravities [20].

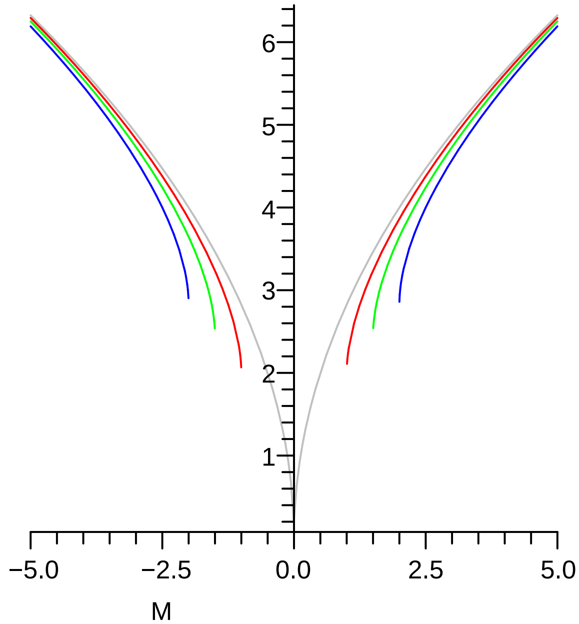

FIG. 2: The normalized entropies $S_{W'}(|\hat{\Omega}|2\pi l_{ren}/4G\hbar)^{-1}$ vs. M for various values of $|J|/l$=0 (grey), 1 (red), 1.5 (green), 2 (blue) [top to bottom] ($l_{ren} = G = |\hat{\Omega}| = 1$).

Regarding the black hole entropy, it has been computed as

$$S_W = \hat{\Omega}\frac{2\pi r_+}{4G\hbar} \tag{24}$$

from the Wald's entropy formula [17, 18, 19]. But this is problematic for $\hat{\Omega} < 0$, though it satisfies the first law (10), since $\delta S_W \leq 0$ from the area theorem which works in this case also due to (19). So, I have recently proposed the modified entropy

$$S_{W'} = |\hat{\Omega}|\frac{2\pi r_+}{4G\hbar}, \tag{25}$$

which agrees with the CFT result as well [11, 12]. Then, I need to consider the modified temperature ${T_+}' = sign(\hat{\Omega})\, T_+$ in order to satisfy the first law

$$\delta M = \Omega_+ \delta J + {T_+}'\delta S_{W'}. \tag{26}$$

The negative temperature ${T_+}'$ for $\hat{\Omega} < 0$ is consistent with the upper bound of mass $M \leq J/l$. The whole behaviors of the temperature can be easily captured in the entropy, as a function of M and J (Fig.2), using (5) and (22):

$$S_{W(new)} = |\hat{\Omega}|\frac{2\pi l_{ren}}{4G\hbar}\sqrt{(4GM/\hat{\Omega})\left[1+\sqrt{1-(J/Ml_{ren})^2}\right]}. \tag{27}$$

As can be observed in Fig.2, this system provides an unusual realization of the negative temperature which does not occur in the usual spin systems.

III. HAWKING TEMPERATURES FROM THE GREEN FUNCTIONS

The Hawking temperature can be fundamentally determined by the periodicity of the thermal Green functions [21]. In the usual black hole systems this agrees with the periodicity for a regular Euclideanized metric at the event horizon r_+. Actually, the Hawking temperature for the BTZ metric has been determined in this way and found to be the same as T_+ of (8) [22]. So, according to the widespread belief that Hawking temperature be uniquely determined by the metric, the new Hawking temperatures which *never* agree with the usual temperature might be considered as unphysical ones. But in this section I show that this is *not* true in general, like as in the systems that I have introduced in Sec. II: There were some "loopholes" in the usual analyses which were unimportant for the ordinary black holes.

To this end, I first note that the Hartle-Hawking Green function for a scalar field in the background metric (1) is given by [I follow the approach of Ichinose-Satoh in Ref. [22]][3]

$$-iG_{BH}(x,x') = \hbar(4\pi l)^{-1}\sum_{n=-\infty}^{\infty}(z_n^2-1)^{-1/2}[z_n+(z_n^2-1)^{1/2}]^{1-\lambda}, \tag{28}$$

where x, x' are the points in the four dimensional embedding space[4] and

$$\begin{aligned} z_n(x,x') - i\varepsilon \;=\; d_H^{-2}\Big[&\sqrt{r^2-r_-^2}\sqrt{r'^2-r_-^2}\,\cosh\left(r_- l^{-2}\Delta t - r_+ l^{-1}\Delta\phi_n\right) \\ &-\sqrt{r^2-r_+^2}\sqrt{r'^2-r_+^2}\,\cosh\left(r_+ l^{-2}\Delta t - r_- l^{-1}\Delta\phi_n\right)\Big] \end{aligned} \tag{29}$$

with $d_H^2 = r_+^2 - r_-^2,\ \Delta t = t - t',\ \Delta\phi_n = \phi - \phi' + 2n\pi$, and an infinitesimal positive imaginary part $i\varepsilon$ [the number λ is a positive number which depends on the mass of the scalar field or the couplings [22]]. Here, it important to note that z_n, and so G_{BH}, is symmetric under $r_+ \leftrightarrow r_-$ interchange; this would be a natural consequence of the symmetry in the metric (1) itself. Then, the Green function on the Euclidean black hole geometry with the Euclidean time $\tau = it$ and the "Euclidean" angle $\varphi = -i\phi$ for $r_- \neq 0$ is

$$G_{BH}^{Eucl}(\Delta\tau,\Delta\varphi;r,r') = iG_{BH}(\Delta t,\Delta\phi;r,r')\big|_{\substack{\Delta\tau=i\Delta t\\ \Delta\varphi=-i\Delta\phi}}. \tag{30}$$

The temperature, now, would be determined by comparing with the *thermal* Green function at temperature β^{-1} and with a chemical potential Ω conjugate to angular momentum [$\mathcal{T}$ denotes the Euclidean time ordered product for scalar fields $\psi(x)$, and $\hat{H}$, $\hat{J}$ are the generators of time translation and rotation, respectively],

$$G_\beta^{Eucl}(x,x';\Omega) = \mathrm{tr}\,[\,e^{-\beta(\hat{H}-\Omega\hat{J})}\mathcal{T}(\psi(x)\psi(x'))]/\mathrm{tr}\,[\,e^{-\beta(\hat{H}-\Omega\hat{J})}], \tag{31}$$

which has the following periodicity:

$$G_\beta^{Eucl}(\tau,\varphi,r;\tau',\varphi',r';\Omega) = G_\beta^{Eucl}(\tau+\beta\hbar,\varphi-\Omega\beta\hbar,r;\tau',\varphi',r';\Omega). \tag{32}$$

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[3] For the system of Sec. IIB, the renormalized parameters, $l_{ren}, r_\pm$, are considered, instead.

[4] The extra coordinates are frozen for the system of Ref. [9].

Because the Green function G_{BH} is a function of z_n, one can find, from (29), that G_{BH}^{Eucl} is periodic under the variation, with $(m, n \in \mathbf{Z})$,

$$\delta(\tau/l) = 2\pi l d_H^{-2}(-r_- m + r_+ n),\ \ \delta(\varphi) = 2\pi l d_H^{-2}(r_+ m - r_- n). \tag{33}$$

If one requires that, as $r_- \to 0$, the chemical potential Ω, which being the angular velocity in a rotating black hole, *vanishes*, the fundamental period is determined uniquely as

$$\tau \to \tau + 2\pi\kappa_+^{-1} n, \quad \varphi \to \varphi - 2\pi\kappa_+^{-1}\Omega_+ n \tag{34}$$

with the angular velocity Ω_+ and the temperature $\beta^{-1} = \hbar\kappa_+/2\pi$ as in (8); this is the usual result [22]. But this does not apply to the exotic systems of Sec. IIA: The chemical potential Ω_- does *not* vanish as $r_- \to 0$ but actually it has a "lower" bound $\Omega_- \geq 1/l$ from (13) [11]. So, in this case, the fundamental period is determined uniquely as

$$\tau \to \tau + 2\pi\kappa_-^{-1} m, \quad \varphi \to \varphi - 2\pi\kappa_-^{-1}\Omega_- m, \tag{35}$$

giving the angular velocity Ω_- and the Hawking temperature $\beta^{-1} = \hbar\kappa_-/2\pi$ as in (13), for $x > 0$. For $x < 0$, on the other hand, the positive temperature $\beta^{-1} = -\hbar\kappa_-/2\pi$ is also determined uniquely by considering $(\hat{H}, \hat{J}, \beta) \to (-\hat{H}, -\hat{J}, -\beta)$, in accordance with the negative M and J, from (3). For the system of Sec. IIB with $\hat{\Omega} < 0$, in which the angular velocity Ω_+ vanishes as $r_- \to 0$ though, the temperature is uniquely determined as $\beta^{-1} = -\hbar\kappa_+/2\pi$, which being negative, with the ordinary angular velocity Ω_+ as in (26), as well as the usual temperature $\beta^{-1} = \hbar\kappa_+/2\pi$ for $\hat{\Omega} > 0$. These results agree completely with the CFT analyses [11, 12]. These systems show that the temperature is *not* uniquely determined by the metric, in contrast to the widespread belief.

IV. CONCLUDING REMARKS

So far I have considered the cases which are described by the three-dimensional metric (1), or up to an extra sphere part. But there are also several other higher-dimensional black hole systems, in the literatures, which show negative Hawking temperatures, though not well recognized. The AdS black holes in higher derivative gravities [23] and the phantom (haired) black holes [24] are the examples, in the type of Sec. IIB (see Ref. [14] for the details). The implications of these black holes to the evolution of the Universe filled with the phantom energy would be quite interesting: If I consider the accretion of the phantom energy onto a black hole with "negative" Hawking temperature, the black hole size increase [25], as in the wormhole cases [26] but in contrast to the ordinary black holes with positive Hawking temperatures [27], until a thermal equilibrium with an equilibrium temperature is reached. This equilibrium is actually possible and can occur *before* the catastrophic situations in Ref. [26] if the phantom energy has the negative temperature as claimed in Ref. [25]. Furthermore, the generalized second law of the phantom Universe with a black hole can be satisfied also with the negative Hawking temperature [25]. The details will appear elsewhere.

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